KALMAN FILTER TYPE SEQUENTIAL METHOD WITH STOPPING RULE FOR CALIBRATION OF MEASUREMENT INSTRUMENTS

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CONTENTS

The least squares method (LSM) is a technique widely used in practice for processing experimental data. In the measurement instruments calibration problem LSM is used for the statistical estimation with high accuracy of coefficients in calibration curves. As is known, the use of LSM has limitations. For one thing, values of the arguments $x(k), k = 1, n$ used to plot a relation $y = f(x)$ may be measured with a certain error.

The main relation for results of measurements in this case takes the form

$$y(k) = f[x(k) + \delta_x(k)] + \delta_y(k)$$

In such a situation, the use of LSM gives usually biased results and what is important incorrect estimates of their errors. Standard instruments for calibration (which reproduce the inputs) also have errors, and LSM in that case may lead to erroneous results.

For a linear relation of the error of measurement, most commonly used in practice, the above equation can be written as

$$y(k) = a + bx(k) + \delta_o(k)$$

where $\delta_o(k) = \delta_y(k) + b\delta_x(k)$ is the reduced error. It should be noted that in this case the classical LSM and the familiar statistical formalism can be used to plot a linear function. However, in order to evaluate the errors of results obtained, one should use reduced error characteristics.

PROBLEM FORMULATION

Let the mathematical model of a system be given in the form of an equation

$$y(k) = \varphi^t(k)\theta$$

In statistics, such a model is called a linear regression, and the vector $\varphi(k)$ a regression vector. The output coordinate $y(k)$ is determined by a measuring system

$$z_y(k) = \varphi^t(k)\theta + v(k)$$

This class of model is of interest for the reason that familiar, efficient and simple methods can be used to evaluate the parameter vector θ. However, these methods were developed for the special case where the regression vector $\varphi(k)$ is accurately known. However, if this vector was measured with an error, the existing methods give inaccurate (biased) results and incorrect error estimates.

Our goal in this study was to design an estimation algorithm for a linear regression model with allowance made for the regression vector error.

ALGORITHM FOR THE ESTIMATION OF A LINEAR REGRESSION MODEL WITH ACCOUNT OF THE REGRESSION VECTOR ERROR

To design the sought estimation algorithm, we use an analogy with the Kalman filter that is used for estimating the state of a linear discrete dynamic system

$$x(k+1) = \Phi(k+1,k)x(k) + B(k+1,k)u(k) + G(k+1,k)w(k)$$

$$z_x(k) = H(k)x(k) + v(k)$$

As is known, the optimum discrete KF for estimation of state vector of linear dynamic system is described by the following system of recurrent equations

$$\hat{x}(k/k) = \hat{x}(k/k-1) + K(k)[z_x(k) - H(k)\hat{x}(k/k-1)]$$

$$K(k) = P(k/k-1)H^t(k)x[H(k)P(k/k-1)H^t(k) + R(k)]^{-1}$$

$$P(k/k) = P(k/k-1) - P(k/k-1)H^t(k)$$

$$\times[H(k)P(k/k-1)H^t(k) + R(k)]^{-1}H(k)P(k/k-1)$$

$$P(k/k-1) = \Phi(k,k-1)P(k-1/k-1)\Phi^t(k,k-1) +$$

$$B(k,k-1)D_u(k-1)B^t(k,k-1) + G(k,k-1)Q(k-1)G^t(k,k-1)$$

The linear regression model is presented in the state and measurement equations form as

$$\theta(k+1) = \theta(k)$$

$$z_y(k) = \varphi^t(k)\theta(k) + v(k)$$

The second expression includes the output coordinate error $v(k)$ and the regression vector error $\delta_\varphi(k)$ .Allowing for this, we obtain

$$z_y(k) = [\varphi(k) + \delta_\varphi(k)]^t\theta(k) + v(k) =$$

$$\varphi^t(k)\theta(k) + \delta_\varphi^t(k)\theta(k) + v(k) =$$

$$\varphi^t(k)\theta(k) + \delta_{rd}(k)$$

$$\delta_{rd}(k) = \delta_{\varphi}^{t}(k)\theta(k) + v(k)$$

In what follows, the measurement error $z_y(k)$ is expressed in terms of the reduced error. The reduced error variance is

$$D_{\delta_{rd}}(k) = \sigma^2 + \theta^t(k) D_{\varphi}(k)\theta(k)$$

Applying optimum linear KF with parameters

$$\Phi(k+1,k) = I; \; Q(k) = 0 = E[w(k)w^t(j)];$$

$$H(k) = \varphi^t(k); \quad E[v(k)v^t(j)] = R(k) = \sigma^2$$

to linear regression model we obtain

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)[z_y(k) - \varphi^t(k)\hat{\theta}(k-1)$$

$$K(k) = \frac{P(k-1)\varphi(k)}{\varphi^t(k)P(k-1)\varphi(k) + \hat{\theta}^t(k-1)D_\varphi(k)\hat{\theta}(k-1) + \sigma^2}$$

$$P(k) = P(k-1) - \frac{P(k-1)\varphi(k)\varphi^t(k)P(k-1)}{\sigma^2 + \varphi^t(k)P(k-1)\varphi(k) + \hat{\theta}^t(k-1)D_\varphi(k)\hat{\theta}(k-1)}$$

The set of equations represent a recurrent procedure and makes it possible to estimate the linear regression model with allowance made for the error in regression vector $\varphi(k)$

ALGORITHM FOR ESTIMATION OF THE CALIBRATION CURVE COEFFICIENTS REGARDING ERRORS IN THE INPUTS

A measuring instrument is calibrated in measurement engineering by means of a high-precision standard instrument that reproduces standard signals for particular measurement intervals, which pass to the instrument and produce output signals that define the calibration characteristic.

The calibration characteristic of the measurement instrument is described adequately by a second-order polynomial:

$$y(p) = a_o + a_1 p + a_2 p^2$$

The measurement equation is written as

$$z_y(k) = a_o + a_1 p(k) + a_2 p^2(k) + \delta_y(k), \quad k = \overline{1, n}$$

The classical form of LSM can be used for estimating the coefficients of calibration polynomial. In thiz case it was assumed that values of arguments $p(k)$ (values generated by standard setting devices) were known exactly.

Since the arguments $p(k)$ are reproducible with an error, the use of LSM may give rise to inaccurate (biased) results and, what is important, may provide incorrect estimates of their errors. To improve data processing in this case, we can use the proposed recurrent estimation algorithm that takes account of the input variable error. We represent the calibration characteristic in vector form:

$$y(k) = \varphi^{t}(k)\theta$$

$$z_{y}(k) = \varphi^{t}(k)\theta + \delta_{y}(k), k = \overline{1, n}$$

where $\varphi^t(k) = |1, p(k), p^2(k)|$ is the input vector (regression vector), $\theta^t = |a_o, a_1, a_2|$ is the vector of unknown (to be estimated) parameters. Since in this case

$$D_\varphi(k) = \begin{vmatrix} 0 & 0 & 0 \\ 0 & D_p(k) & K_{p^2,p}(k) \\ 0 & K_{p^2,p}(k) & D_{p^2}(k) \end{vmatrix}$$

AN EXPERIMENTAL EXAMPLE

As an example, we consider the problem of calibration of a differential pressure gauge using standard setting devices (piston gauges). The calibration characteristic of the differential gauge is described as a second- order polynomial.

The following initial conditions and input data for the experiment and calculations are taken.

The standard pressures were specified in the range $0 \le p(k) \le 1600$ bar with a step of 100 bar provided by an MP-2.5 piston manometer of accuracy class 0.02.

The measurements were made with a Sapfir-22DD differential pressure transducer with relative error % 0.5 and measurement range 0–1600 bar. The output signal from the transducer is an electric voltage measured in mV. Experimental results are given in the Table 1.

Table 1. Calibration Measurements Results

Measurement №	p, bar	y, mV
1	0	0.0051
2	100	545.0231
3	200	1192.3651
4	300	1942.8624
5	400	2798.7251
6	500	3758.5748
7	600	4821.0851

8	700	5990.285
9	800	7261.674
10	900	8635.6752
11	1000	10113.8051
12	1100	11697.1894
13	1200	13384.1651
14	1300	15181.0295
15	1400	17076.9348
16	1500	19073.5267
17	1600	21172.8851

The errors of the piston gauges and differential pressure gauge are normally distributed with zero mathematical expectation and standard deviations, respectively, are

$\sigma_p(k) \approx 0.1$ bar; $\sigma(k) \approx 35.3$ mV.

The values of calibration coefficients determined from proposed algorithm are given in Fig. 1 and the variances of the corresponding estimation errors are shown graphically in Fig.2.

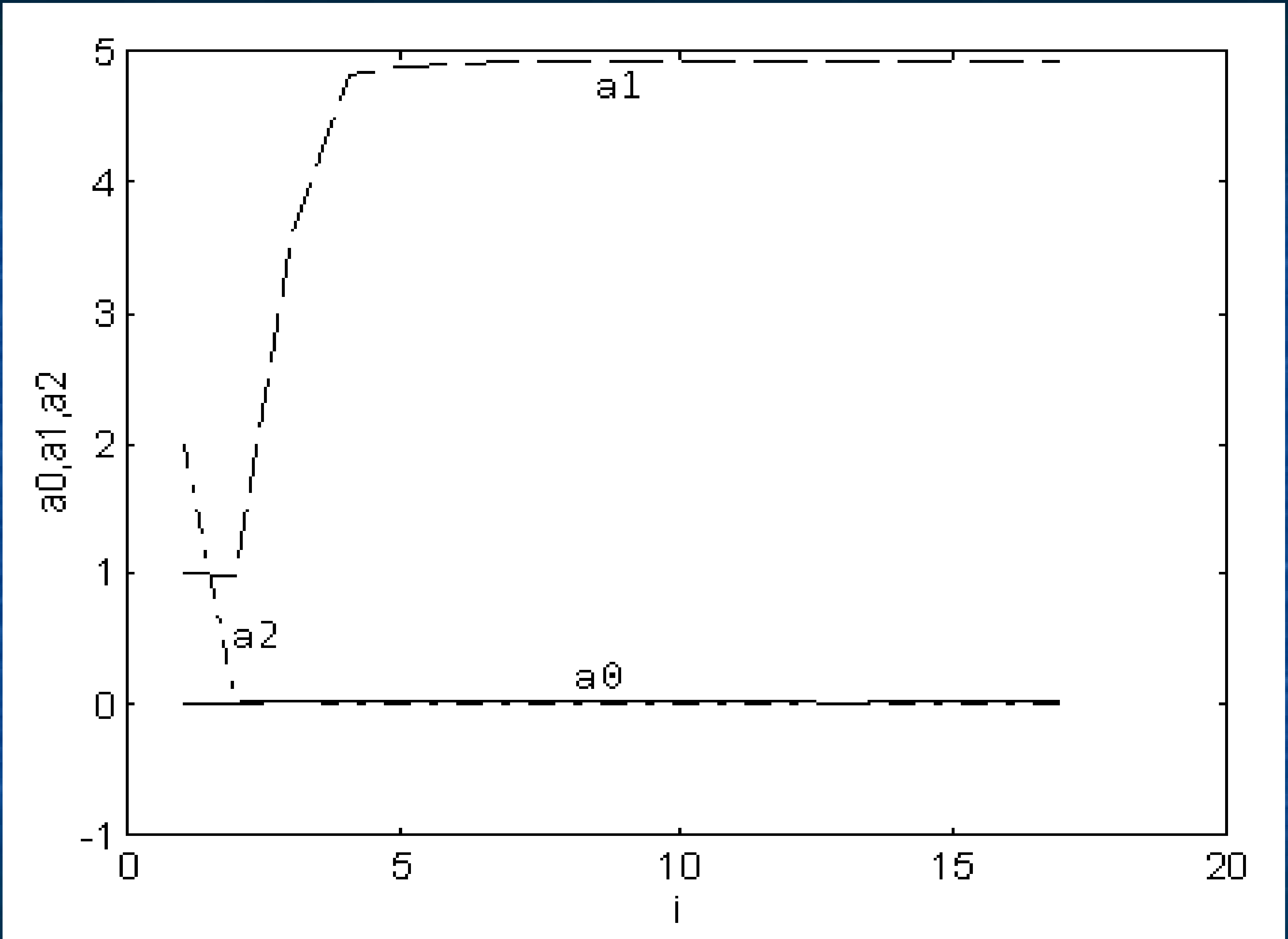

Fig.1.Variations in the calibration coefficients $\hat{a}_0, \hat{a}_1, \hat{a}_2$

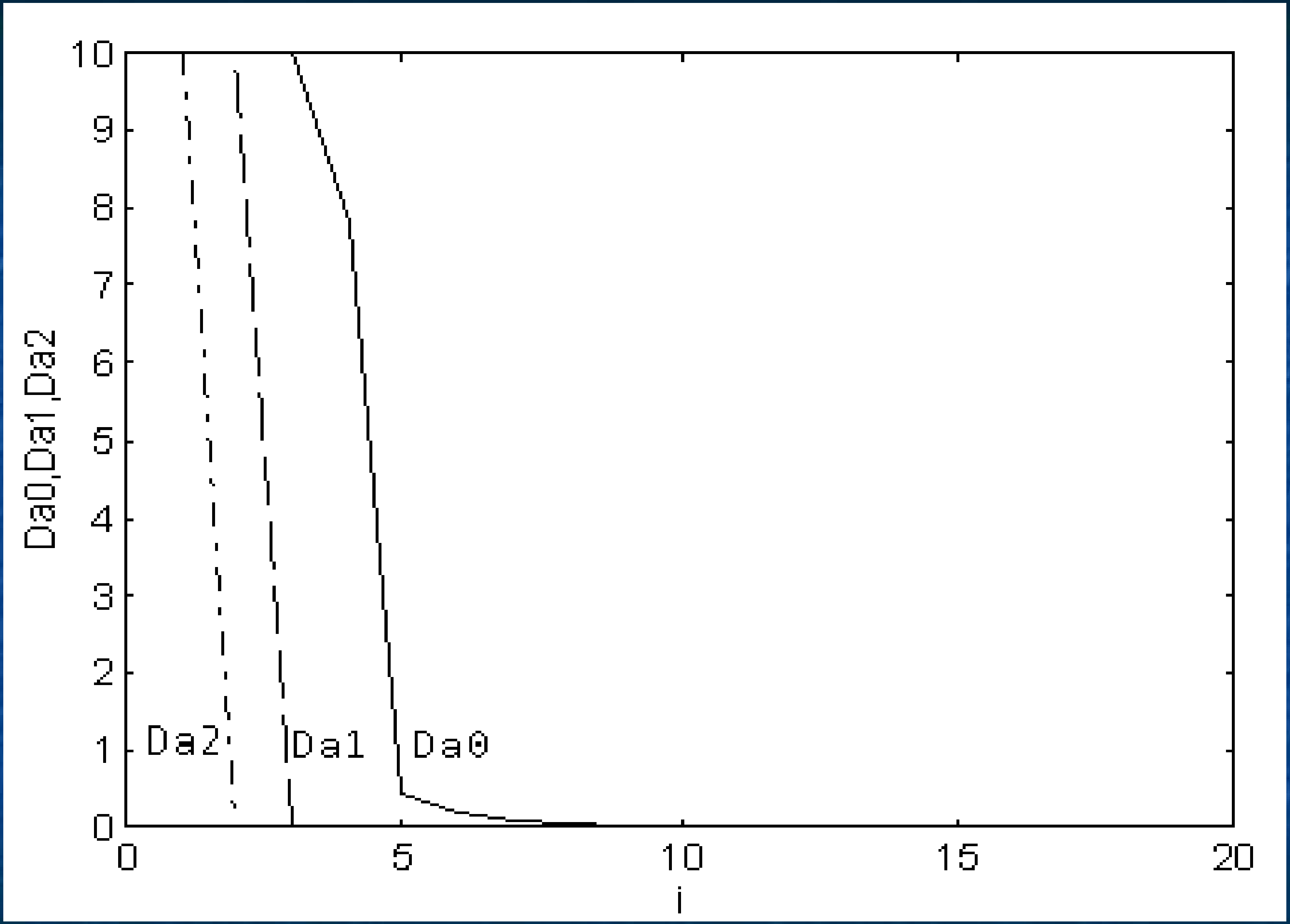

Fig.2. Variations in the estimation error variances $D_{\hat{a}_0}, D_{\hat{a}_1}, D_{\hat{a}_2}$

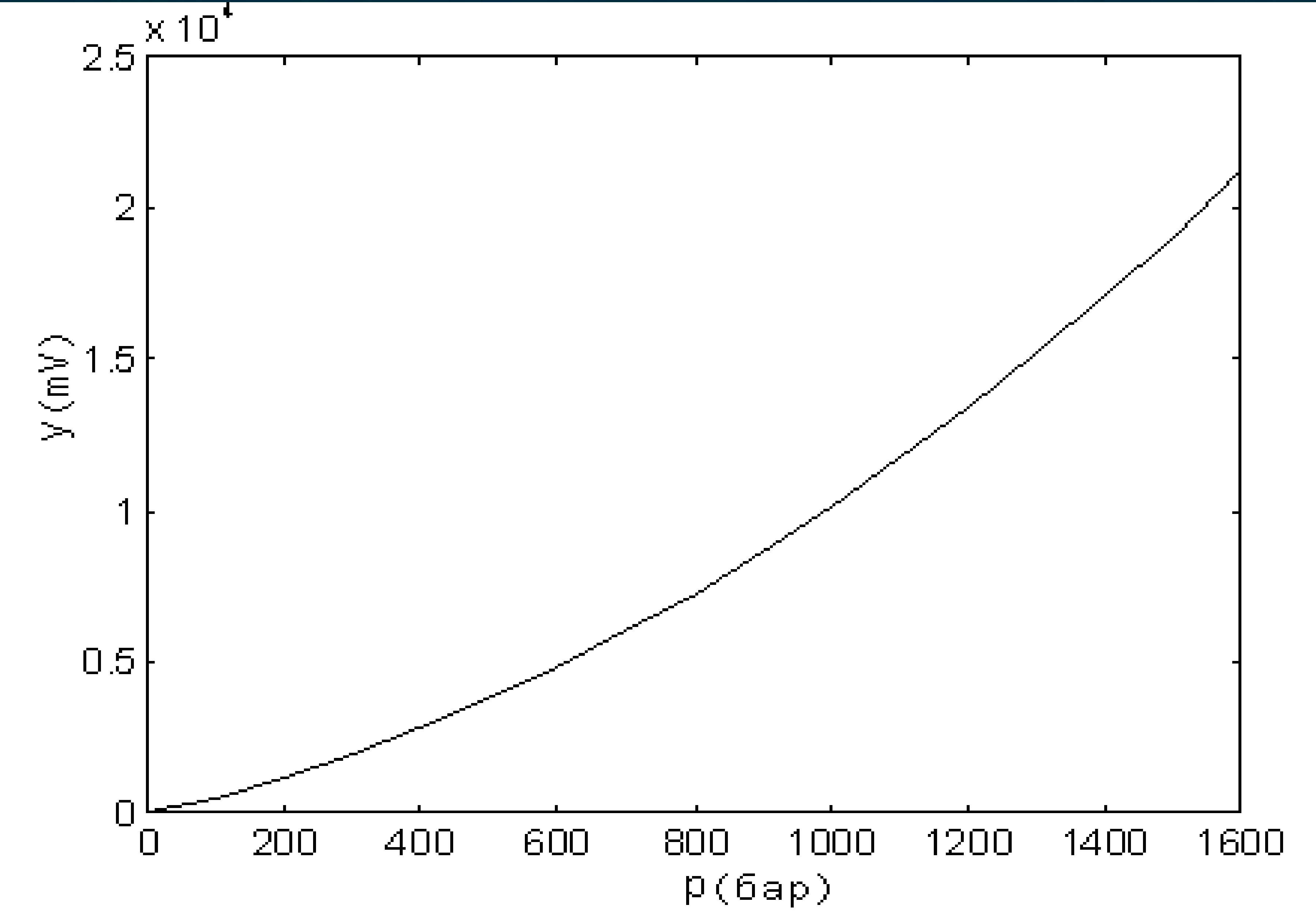

Fig.3. Calibration curve obtained with proposed algorithm

These results demonstrate high accuracy and rapid convergence for the estimation algorithm.

After the coefficients $\hat{a}_o, \hat{a}_1, \hat{a}_2$ have been determined, the polynomial

$$y(k) = \hat{a}_o + \hat{a}_1 p(k) + \hat{a}_2 p(k)$$

can be used as a calibration curve for a differential pressure gauge. Under actual operating conditions, measurements $z_y(k)$ from the primary transducer are processed by the microprocessor of the differential pressure transducer to solve the inverse problem, that is roots of calibration equation are found

$$\hat{p}_{1,2}(k) = \frac{-\hat{a}_1 \pm \sqrt{\hat{a}_1^2 - 4\hat{a}_2(\hat{a}_o - z_y(k))}}{2\hat{a}_2}$$

and the root $\hat{p}_1(k)$ is taken as the measured pressure estimate. A verification test was carried out via above expression using presented in the Table 1 calibration experiment results. The obtained calibration values $\hat{p}(k)$ and absolute and relative calibration errors are presented in the Table 2.

Table 2. Calibration errors in using the proposed Algorithm

Estimates, $\hat{p}(k)$ bar	Absolute Error, Δ_{abs}, bar	Relative Error, $\Delta_{rel}, \%$
-0.0026	0.0026	100
100.2274	0.2274	0.2269
200.1496	0.1496	0.0748
299.9868	0.0132	0.0044
400.0118	0.0118	0.0029
500.0303	0.0303	0.0061
599.9253	0.0747	0.0124

700.0588	0.0588	0.0084
800.0344	0.0344	0.0043
899.9164	0.0836	0.0093
999.8229	0.1771	0.0177
1099.8	0.1821	0.0166
1199.8	0.2100	0.0175
1300.1	0.0847	0.0065
1400.1	0.0933	0.0067
1499.9	0.0605	0.0040
1599.7	0.2567	0.016

Comparison of the Classical Recurrent LSM and the new Algorithm

Parameters	Classical LSM	New algorithm
$\hat{a}_0$	0.0229	0,0182
$\hat{a}_1$	4.9166	4,9165
$\hat{a}_2$	0.0051	0,0052
$D_{\hat{a}_0}$	0.0891	0.0918
$D_{\hat{a}_1}$	0.001396	0.001403
$D_{\hat{a}_2}$	0.0000000083	0.0000000084

Absolute Errors (bar), when the new method was used	Absolute Errors (bar), when conventional method was used
0.0026	0.0036
0.2274	0.2249
0.1496	0.1460
0.0132	0.0175
0.0118	0.0069
0.0303	0.0349
0.0747	0.0804

0.0588	0.0526
0.0344	0.0280
0.0836	0.0901
0.1771	0.1840
0.1821	0.1891
0.2100	0.2172
0.0847	0.0873
0.0933	0.0858
0.0605	0.0681
0.2567	0.2643

GENERATION OF CALIBRATION PROCEDURE STOPPING RULES

The results of the general mathematical theory of optimal stopping rules, have not enjoyed any appreciable application in the generation of stopping rules in parametric identification problems.

In application to multidimensional parametric identification problems the stopping rule based on comparing the statistics of the differences between two successive estimates is introduced.

That rule can be used for stopping the calibration procedure of measurement instruments through following statistic:

$$r_i^2 = (\hat{\theta}_i - \hat{\theta}_{i-1})^T D_{\Delta\theta_i}^{-1} (\hat{\theta}_i - \hat{\theta}_{i-1}) \leq \varepsilon,$$

The statistic r_i^2 has a $\chi_{\beta_1}^2$ distribution with *n* degrees of freedom. The estimation process is stopped when

$$r_i^2 < \chi_{\beta_1}^2$$

since further measurements yield insignificant improvement in the calibration characteristic.

This stopping rule can be used to make a timely decision to stop the calibration process, and it does not require large computational expenditures. The covariance matrix $D_{\Delta\theta_i}$ of the discrepancy between two successive estimates $\hat{\theta}_{i-1}$ and $\hat{\theta}_i$ may be written in the form

$$D_{\Delta\theta_i} = P_{i-1} - P_i$$

The estimates obtained at the stopping moment will determine the calibration characteristic. As a consequence, the calibration of the measurement instrument is performed with the required accuracy on the minimum number of calibration measurements.

AN EXPERIMENTAL EXAMPLE

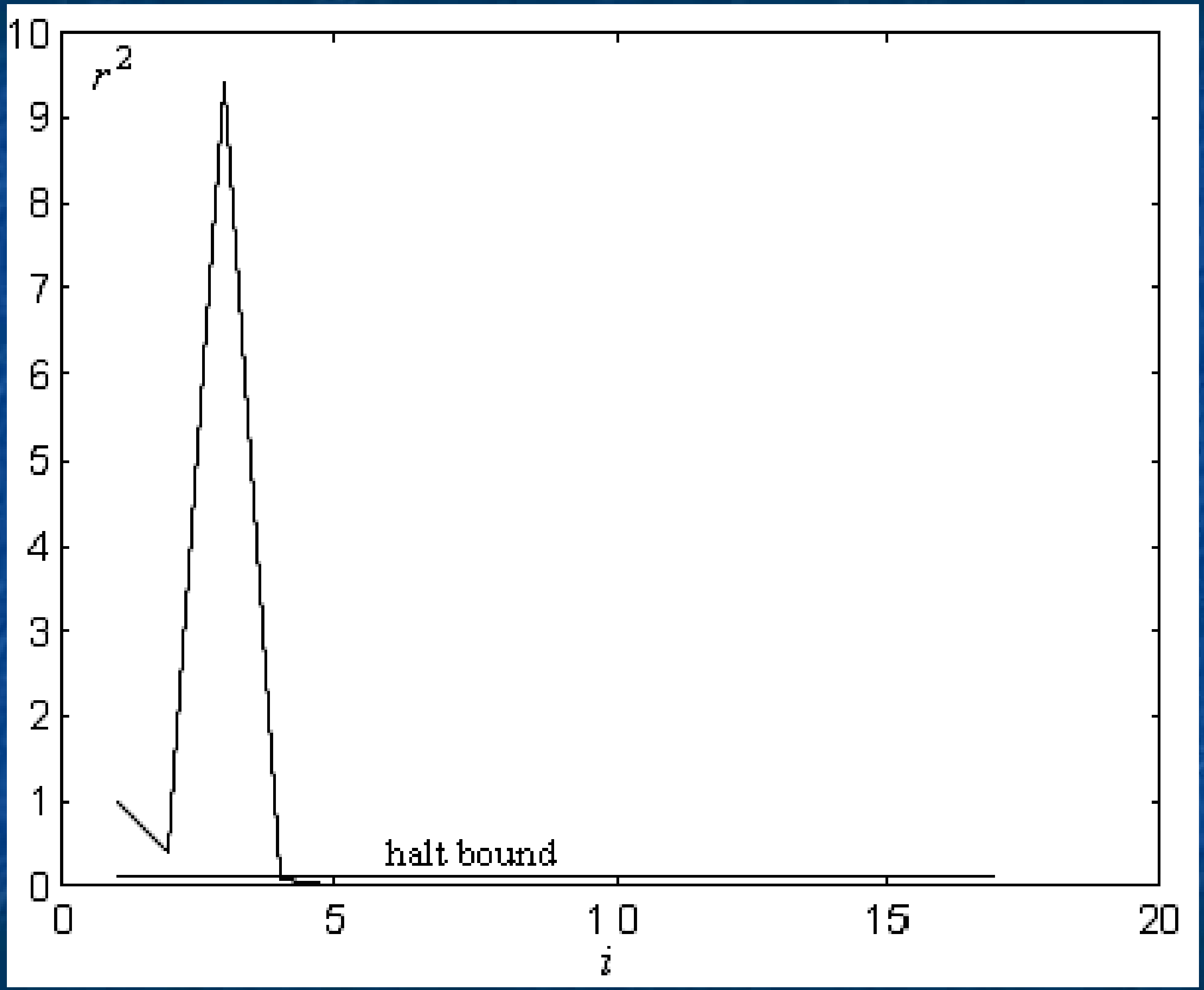

Fig. 4. Behavior of statistic r_i^2 when proposed calibration algorithm was used

As it is seen from these results, after the fifth step of estimation, according to the decision rule, the calibration should be stopped, because further measurements lead only to a slight improvement in the calibration curve and are considered undesirable. The proposed algorithm provides the required calibration accuracy with a smaller number of measurements.

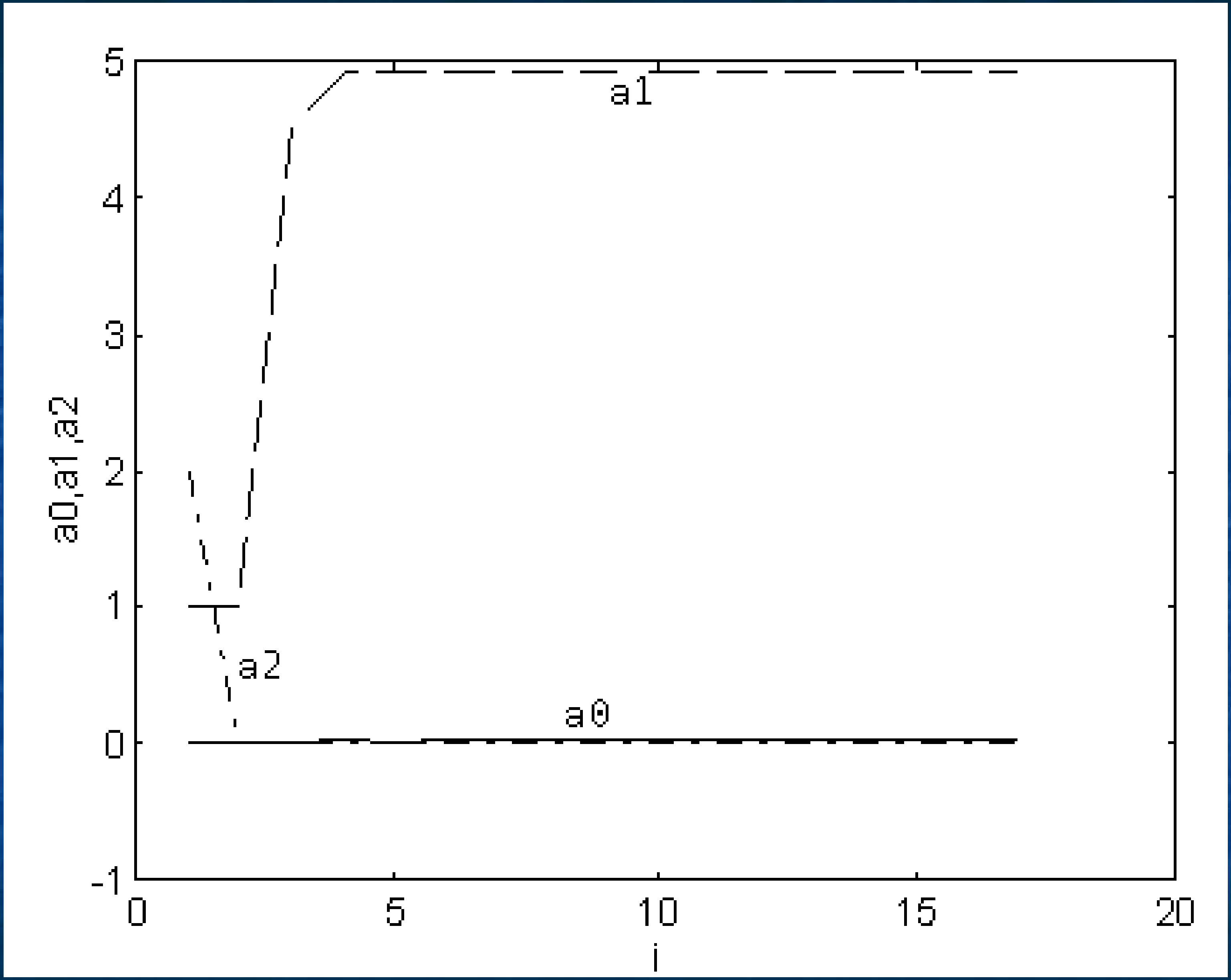

Fig.4. Variations in the calibration coefficients

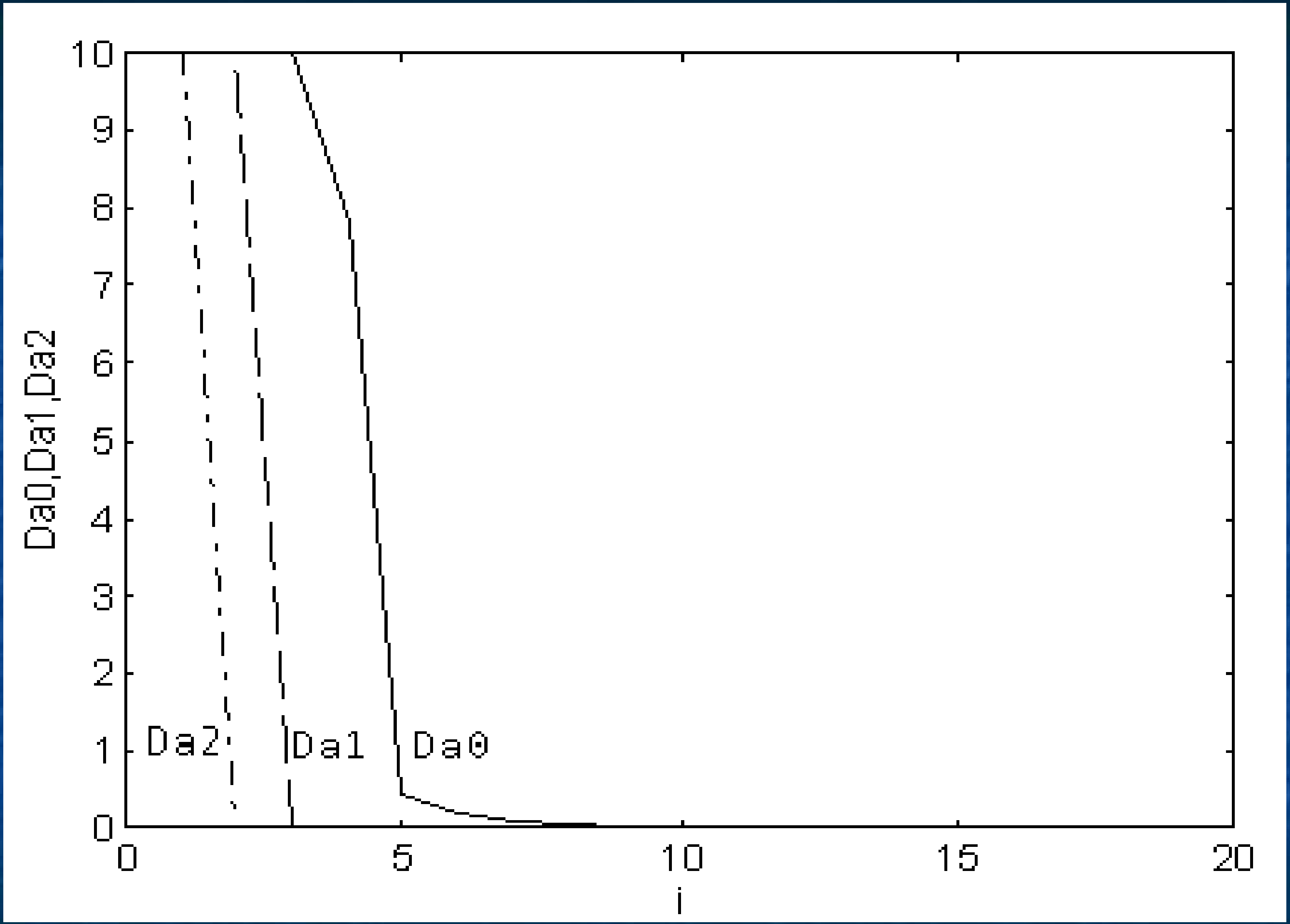

Fig.2. Variations in the estimation error variances $D_{\hat{a}_0}, D_{\hat{a}_1}, D_{\hat{a}_2}$

CONCLUSION

- A sequential method with stopping rule for calibration of measurement instruments is presented.
- The presented method differs from existing calibration methods in that it incorporates the errors in the inputs.
- The algorithm improves the accuracy of the estimates and gives reliable evaluations of their errors.

CONTINUED

- A new approach has been proposed for the generation of stopping rule in calibration characteristics identification problem. As a consequence, the calibration of the measurement instrument is performed with the required accuracy on the minimum number of calibration measurements.
- The reduction of the number of calibration measurements reduces the economical and time expenses. That is particularly important when the cost of each measurement is high.

THANK YOU FOR YOUR ATTENTION